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The report discusses the specific problem of controlling a monopoly when market demand is used as the basis for measuring benefits. The control tool used to achieve the regulators objectives is called an "optimal subsidy function." It is shown that while such an optimal function always exists for the one good case, when more than one good is involved existence requires equality of the market cross derivatives of demand with respect to price.

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OPTIMAL SUBSIDY FUNCTIONS

SECTION I

INTRODUCTION

The "textbook" analysis of monopoly regulation when average costs are decreasing has been summarized as

> Public utility regulation of the monopoly would set its price down to the intersection of demand with long-run average cost; this wipes out excess profit, more important, it brings price closer to the marginal cost level. . .where marginal social costs and benefits are appropriately balanced.
>
> Ideally, price should be forced all the way down to marginal cost. . ., with the chronic loss covered by permanent government (lump-sum) subsidy (4:501).

There is, however, a fundamental problem in monopoly regulation which seems to have been ignored in conventional analysis. The problem is that the regulators may not know the monopolist's cost function, and it would, therefore, be difficult to set the regulated price equal to marginal cost even if the consumer's demand function were known.

In this paper, I consider the basic problem of deriving "an optimal subsidy function," defined to be a rule for calculating profits which motivates a monopolist to produce at the output level(s) where price equals marginal cost, but which does not depend on the cost conditions of the monopolist. It is assumed that the regulators do not know the monopolist's cost function, but do have a knowledge of demand.

Partial Equilibrium Analysis

Two alternative partial equilibrium constructions will be used, both of which contain the assumption that the prices of all other commodities produced in the economy are constant. These other goods can, therefore, be collapsed into a composite commodity with a composite price that will be absorbed into the applicable functional forms throughout the analysis.

I will call the first approach the Marshallian Subsidy Function in view of its compatibility with the partial equilibrium tradition of Marshall. The aggregate demand function faced by the monopolist is of the aggregate Marshallian variety, that is, the money income of each of the individual consumers is held constant throughout the analysis. Thus, the consumers do not pay the subsidy nor do they receive any part of the profits of the monopolist who spends these profits on other goods produced in the economy.

There are several problems with this approach. One is that the payment of a subsidy out of general taxes may lead to second best problems. A second problem is that the income effects that are embodied in the Marshallian demand function will be activated when the price is varied by the monopolist. These income effects will affect the distribution of income in a manner that is unlikely to be optimal. In addition, when more than one good is produced by the monopolist, this subsidy function does normally not exist.

Thus, I propose an alternative approach called the Income Compensation Subsidy Function. This approach uses the income compensation function pioneered by Hurwicz and Uzawa (1:114-148). The existence of this function depends on utility maximizing behavior by the consumer. The consumer pays the subsidy out of personal income, and this enables one to explicitly consider the distribution of real income among consumers when constructing this subsidy function. However, it is again assumed that the monopolist spends all the profits on other goods produced in the economy.

I first compare the two approaches in Section II under the assumption that one good is produced by the monopolist. This is followed in Section III by a two-goods analysis.

SECTION II

ONE-GOOD ANALYSIS

Marshallian Subsidy Function

The monopolist is assumed to be a profit maximizer who produces a single output x under cost conditions represented by C(x). The cost function is unknown to the regulators. The aggregate demand curve faced by the monopolist, and known by the regulators, is of the form

$$x = X(p)$$

where p is the price per unit of output.

For any subsidy function, H(p), selected by the regulators, the monopolist's problem is to solve

$$\max_{p} \{pX(p) + H(p) - C(X(p))\}.$$

The first order condition for this problem is

$$(p - C')X' + X + H' = 0.$$

As X' is non-vanishing, the regulators can achieve the social objective of price equal to marginal cost if, and only if, the optimal subsidy function $\overset{*}{H}$ is structured such that

$$\overset{*}{H}{}' = -X(p).$$

Thus, one obtains via integration

$$\overset{*}{H} = -\int_{\bar{p}}^{p} X(\xi)d\xi + A$$

where $\bar{p}$ and A are jointly set by the regulators.

Therefore, the monopolist's problem becomes

$$\max_{p} \{pX(p) - \int_{\bar{p}}^{p} X(\xi)d\xi + A - C(X(p))\}.$$

One can then show that the monopolist's maximization of this function results in price being equated to marginal cost as is desired. In addition, for this one-good analysis the function $\overset{*}{H}$ always exists, but as we will see, this is not true when more than one good is being controlled.

Income Compensation Subsidy Function

The income compensation function, $\mu(p/p^o, m^o)$, determines the minimum income required by the consumer when he faces a price p, to achieve the same utility level he could enjoy (by maximizing behavior) under a price income situation (p^o, m^o).[1] The function thus constrains the consumer to the indifference curve obtained in some base situation (p^o, m^o). An important property of the income compensation function is that (1: 120-121)

$$\frac{\partial\mu(p/p^o, m^o)}{\partial p} = D\,(p, \mu(p/p^o, m^o)). \tag{1}$$

In that the function μ holds utility constant at the level associated with parametric situation (p^o, m^o), $D\,(p, \mu(p/p^o, m^o))$ is a hicks compensated demand function. Furthermore, in that μ is measured in terms of money income, the function D is the observable Marshallian uncompensated demand function. By using partial differential equation (1) and the definitional boundary condition,

$$\mu(p^o/p^o, m^o) \equiv m^o,$$

it is possible to obtain the income compensation function. However,

it is necessary for the regulators to know the demand function(s) of individual consumer(s) and not simply the aggregate demand curve as was required in the Marshallian subsidy function analysis.

The approach taken is to define the subsidy function, and then show that the social objective is achieved. We assume that there is a single utility maximizing consumer with base income $\bar{m}$ who possesses a Marshallian demand function for the single good x equal to $D(p,\bar{m})$, where p is the price of the one good under consideration. The subsidy function $H^c(p)$, is defined as

$$H^c(p) = \bar{m} - \mu(p/p^o, m^o)$$

and does not depend on the unknown cost function of the producer. One is reminded that the subsidy is paid by the consumer to the monopolist. The subtraction of the subsidy from $\bar{m}$ transforms the consumer's demand function into a Hicks compensated demand function of the form

$$D(p, \mu(p/p^o, m^o)) .$$

Therefore, producer's problem is to solve

$$\max_p \{pD(p, \mu(p/p^o, m^o)) + H^c(p) - C(D(p, \mu(p/p^o, m^o)))\}$$

which has an associated first order condition,[2]

$$(p - C')[D_1 + D_m D] = 0 .$$

As the sum, $D_1 + D_m D$, is the slope of the compensated demand function, it is non-zero and the monopolist maximizes profits when price is equated to marginal cost. It should also be noted that

the regulators can set the parameters of the income compensation function (p^o, m^o) equal to those values which place the consumer at the socially desired utility level without distorting the producer's incentive to equate price and marginal cost. The distributional issue can be dealt with at the same time that allocative efficiency is achieved.

It is illuminating to compare the two approaches graphically. The Marshallian Subsidy Function approach is depicted in Fig. 1.

MARSHALLIAN SUBSIDY FUNCTION

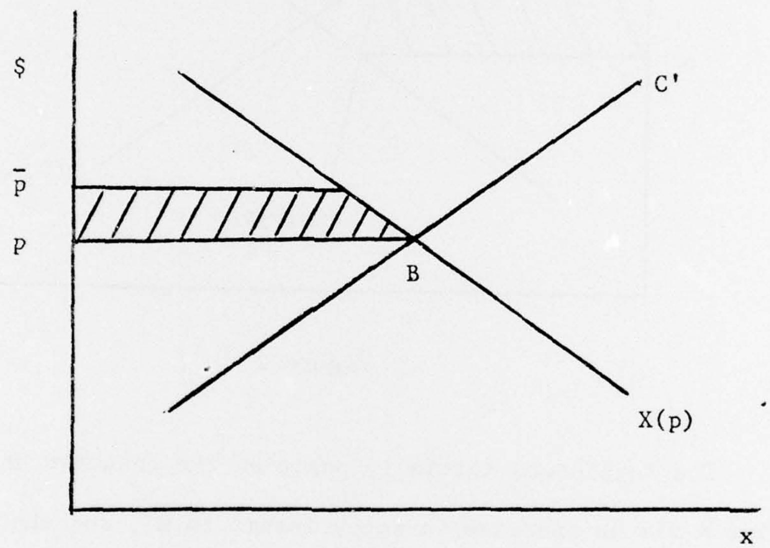

Figure 1

The cross hatched area (less the constant A) equals the subsidy earned by the monopolist when $\bar{p}$ equals the parameter of the subsidy function selected by the regulators. Note that the income effect that is embodied in the aggregate demand curve is

permitted to completely work itself out when point B is selected by the monopolist.

Fig. 2 reviews the Income Compensation Function approach to the optimal subsidy function.

INCOME COMPENSATION SUBSIDY FUNCTION

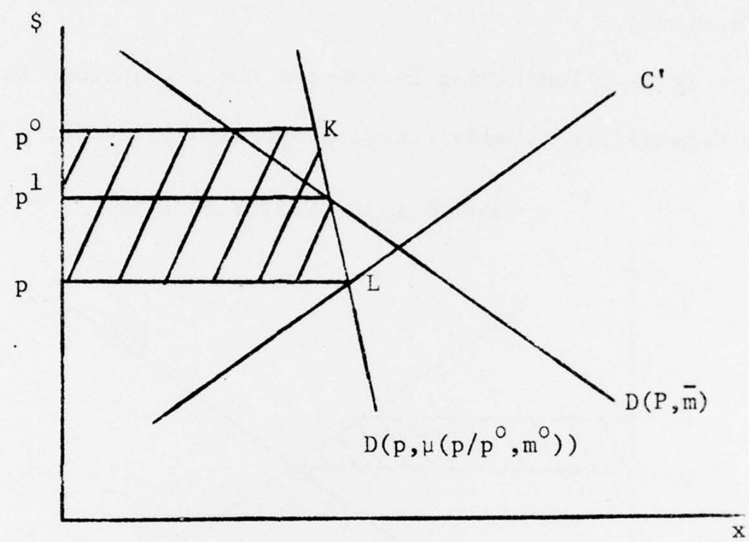

Figure 2

The regulators initially position the consumer at point K via an increase in money income to m^o, and the announcement that the income compensation function parameter price is p^o. It would also be possible to position the consumer on this same indifference curve without changing money income through the use of the parameter price p^1. The cross hatched area which is the consumer's compensating variation for the move from point K to point L is also the subsidy received by the monopolist (less the difference between consumer income $\bar{m}$ and m^o).

The output selected by the monopolist depends on the parameters (p^o, m^o). If the consumer is placed on a different compensated demand function, a point other than L would be selected by the monopolist, although an outcome would be selected in which price equals marginal cost as is socially desired.

It should also be noted that the parameter $\bar{p}$ of the Marshallian approach, and the parameter p^o of the income compensation function approach need not bear any relationship to the price that the monopolist was originally charging.

SECTION III

TWO-GOODS ANALYSIS

Marshallian Subsidy Function

The monopolist will again be assumed a profit maximizer who now produces two commodities which have associated aggregate demand "curves,"

$$x^j = X^j(p_1,p_2) \quad j = 1,2 .$$

When any subsidy function, $H(p_1,p_2)$, is announced by the regulators, the monopolist's problem is to solve

$$\operatorname*{Max}_{p_1,p_2} \{p_1X^1(p_1,p_2) + p_2X^2(p_1,p_2) + H(p_1,p_2) - C(X^1,X^2)\}.$$

The first order conditions for the problem can be written in partitioned matrix form as

$$\left(\begin{array}{c|cc} H_1+X^1 & X_1^1 & X_1^2 \\ H_2+X^2 & X_2^1 & X_2^2 \end{array}\right)\left(\begin{array}{c} 1 \\ \hline p_1-C_1 \\ p_2-C_2 \end{array}\right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

or

$$\left(G \mid D\right)\left(\begin{array}{c} 1 \\ \hline \gamma \end{array}\right) = 0 .$$

The determinant of the submatrix D is assumed to be non-zero in demand theory (for example, in order to obtain the inverse demand functions (3:377)). This assumption is used to prove that a subsidy function $\overset{*}{H}(p_1,p_2)$ which achieves equality of price and marginal cost for the two goods will normally not exist.

Proposition 1: The regulators will achieve the social objective of price equal to marginal cost for both goods if, and only if, $\overset{*}{H}_j = -X^j$; therefore, $\overset{*}{H}$ will exist if, and only if, $X_k^j = X_j^k$, k,j = 1,2.

To prove the first part of the proposition, assume that G = 0. The remaining part of the partitioned matrix can then be written

$$D\gamma = 0 \ . \qquad (2)$$

This equation system can have a non-trivial solution for γ if, and only if, the determinant of D is equal to zero, but as indicated above this is assumed to be non-zero. The only solution to (2), therefore, is the solution, $\gamma = 0$, i.e., $p_j = C_j$ (j = 1,2).

Conversely, if D is non-singular and $\gamma = 0$, then G = 0 which implies that

$$\overset{*}{H}_j = -X^j \ , \quad j = 1,2 \ .$$

and therefore the first part of the proposition is proved.

In order to prove the second part of the proposition, an important result from the theory of line integrals will be exploited. Under appropriate regularity conditions, the following can be shown to be equivalent.[3]

(1) $\int_B \overset{*}{H}_1 d\xi_1 + \int_B \overset{*}{H}_2 d\xi_2$ is independent of the path of integration B where B is any piecewise smooth curve joining two points.

(2) There is a smooth function, $\overset{*}{H}(p_1,p_2)$, such that the gradient of $\overset{*}{H}$, $\nabla\overset{*}{H} = (\overset{*}{H}_1, \overset{*}{H}_2)$.

(3) $\overset{*}{H}_{12} = \overset{*}{H}_{21}$.

In that $\overset{*}{H}_{12} = \overset{*}{H}_{21}$ is equivalent to $X_k^j = X_j^k$, $j,k = 1,2$, when the first part of the proposition is satisfied, the second part of the proposition is proved QED.

If one assumes that the integrability condition, $X_k^j = X_j^k$, is satisfied, then one can solve for $\overset{*}{H}(p_1,p_2)$ by integration over a convenient path. We will choose a path of integration in which prices are varied from $\overline{p}_j$, $j = 1,2$, one at a time. We therefore obtain

$$\overset{*}{H}(p_1,p_2) = -\int_{\overline{p}_1}^{p_1} X^1(\xi_1,\overline{p}_2)d\xi_1 - \int_{\overline{p}_2}^{p_2} X^2(p_1,\xi_2)d\xi_2 + A,$$

where $\overline{p}_1$, $\overline{p}_2$, and the constant A are parameters of the subsidy function to be chosen by the regulators.

Therefore, the monopolist must solve

$$\max_{p_1,p_2}\{p_1X^1(p_1,p_2) + p_2X^2(p_1,p_2) + \overset{*}{H}(p_1,p_2) - C(X^1,X^2)\}.$$

This can be viewed as a maximization problem which yields the right answer, though it is only under unusual circumstances that the integrability condition would be satisfied.

The proposition implies that if more than one good is being produced by the monopolist, knowledge by the regulators of aggregate demand is normally not sufficient to achieve equality of price and marginal cost. The non-existence of an optimal subsidy function results from the "non-equal" income effects associated with the two goods. This lack of symmetrical income effects implies that the "area" under the demand curves is dependent on the path of integration,

and, therefore, a function associated with this area is not defined. This result also implies that it is not possible to do exact non-local comparative statics welfare economics using the area under the demand curves unless there is an equality of the cross partial derivatives. A necessary condition for this type of welfare economics is that the relevant function which determines changes in welfare exist.

Income Compensation Subsidy Function

One Consumer

At first, I assume there is one consumer who pays the subsidy. Furthermore, the monopolist spends the profits on the other goods produced in the economy. The subsidy function is defined as the consumer's original money income $\bar{m}$ less the income compensation function μ parameterized at the $(p_1^o, p_2^o, \overset{o}{m})$ which places the consumer on the socially preferred indifference curve. Therefore, the subsidy function is of the form

$$H^c(p_1, p_2) = \bar{m} - \mu(p_1, p_2/p_1{}^o, p_2{}^o, m^o)$$

and the profit function of the monopolist is

$$\pi(p_1, p_2) = p_1 D^1(p_1, p_2, \mu) + p_2 D^2(p_1, p_2, \mu) + \bar{m}$$
$$-\mu(p_1, p_2/p_1{}^o, p_2{}^o, m^o) - C(D^1, D^2).$$

Setting the derivatives of the profit function with respect to p_1 and p_2 equal to zero yields

$$\pi_1 = (p_1 - C_1)[D_1^1 + D_m^1 D^1] + (p_2 - C_2)(D_1^2 + D_m^2 D^1) = 0 \tag{3}$$

$$\pi_2 = (p_1 - C_1)[D_2^1 + D_m^1 D^1] + (p_2 - C_2)(D_2^2 + D_m^2 D^2) = 0. \tag{4}$$

The differentiation of compensated demand functions yields the Slutsky compensated derivatives, S_{kj} .

$$S_{kj} = D_j^k + D_m^k D^k \qquad \begin{array}{l} k = 1,2 \\ j = 1,2 \end{array},$$

and (3) and (4) can be written in matrix form as

$$\begin{pmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{pmatrix} \begin{pmatrix} p_1 - C_1 \\ p_2 - C_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

or

$$B\gamma = 0 \; . \tag{5}$$

Matrix B is the submatrix of the entire Slutsky matrix for all goods consumed by this single consumer, and is the matrix associated with a principal minor. From demand theory, we know that this matrix is strictly negative definite, which implies that it has a non-zero determinant. Therefore, the only solution to (5) is $\gamma = 0$, i.e., price equated to marginal cost for each good.

Simultaneously, the consumer's equitable utility level can be achieved through appropriate choice of $(\overset{o}{p}_1, \overset{o}{p}_2, \overset{o}{m})$. The producer will equate price and marginal cost for any choice of these parameters.

Thus, this subsidy function surmounts the integrability problem associated with the Marshallian Subsidy Function (it exists), achieves allocative efficiency through equality of price and marginal cost, and simultaneously provides a way of achieving the distributional objectives. I now generalize the approach to deal with the case where there are many consumers. It is assumed that

the demand functions of each consumer are known.

Many Consumers

Each of n consumers (i = i,...,n) with Marshallian demand functions for the two goods, $D^{ij}(p_1,p_2,m^i)$, j = 1,2, pays the monopolist a subsidy

$$H^{ci} = \bar{m}^i - \mu^i(p_1,p_2/p_1^{oi},\ p_2^{oi},\ m^{oi}).$$

It is important to note that the parameters of the subsidy function can be individualized. Each consumer can be simultaneously facing the same market prices, p_1,p_2, and consumer specific income compensation function price parameters, p_1^{oi} and p_2^{oi}. These price parameters together with m^{oi}, the money income parameter, specify the socially desired level of utility of each consumer.

The monopolist's market response is to the sum of the individual consumer's demand functions. As each consumer's demand function becomes a compensated demand function following the subtraction of the subsidy paid to the monopolist, the monopolist responds to aggregate compensated demand, and his problem is to solve

$$\text{Max } \pi = p_1\sum_i D^{i1}(p_1,p_2,\mu^i) + p_2\sum_i D^{i2}(p_1,p_2,\mu^i) + \sum_i H^{ci} - C(D^1,D^2).$$

The associated first order conditions are

$$\pi_1 = (p_1-C_1)\sum_i(D_1^{i1} + D_m^{i1}D^{i1}) + (p_2-C_2)\sum_i(D_1^{i2} + D_m^{i2}\ D^{i1}) = 0 \quad (6)$$

$$\pi_2 = (p_1-C_1)\sum_i(D_2^{i1} + D_m^{i1}\ D^{c1}) + (p_2-C_2)\sum_i(D_2^{i2} + D_m^{i2}\ D^{i2}) = 0. \quad (7)$$

The terms inside the summation signs adjacent to $(p_j - C_j)$ are the individual Slutsky derivatives, S_{jk}. Thus, (6) and (7) can be written in matrix form as

$$\begin{bmatrix} \sum_i S^i_{11} & \sum_i S^i_{21} \\ \sum_i S^i_{12} & \sum_i S^i_{22} \end{bmatrix} \begin{bmatrix} p_1 - C_1 \\ p_2 - C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

or using notation similar to the one consumer analysis as

$$\sum_{i=1}^{m} B^i \gamma = 0 .$$

As each individual B^i matrix is strictly negative definite, so too is the sum of n strictly negative definite matrices. Therefore, the determinant of ΣB^i is non zero. As γ must equal zero, the monopolist equates price and marginal cost for each commodity.

This approach requires each consumer to pay the monopolist his individualized, H^{ci}, and it is not sufficient that the total subsidy paid, ΣH^{ci}, be correct. There is, therefore, a requirement that the regulators know each consumer's demand function.

SECTION IV

CONCLUSION

I have discussed two alternative approaches to the construction of a subsidy function for use in an environment in which the regulators do not know the monopolist's cost function, but do desire that the monopolist equate price and marginal cost.

The Marshallian Subsidy Function normally does not exist when the monopolist produces more than a single good. In addition, the requirement that the subsidy be paid out of general taxes may lead to distortions elsewhere in the economic system. Furthermore, this approach does provide a way of satisfying the distributional objectives.

The income compensation subsidy function requires the assumption of utility maximization by one or many consumers. The tax problem is dealt with by requiring the consumer(s) to pay the subsidy, and simultaneously, distributional objectives can be achieved.

The approach does, however, require that the individual demand function(s) be known, whereas for the Marshallian subsidy function approach it is sufficient to know the demand curve(s).

FOOTNOTES

[1]For the one-good analysis, all other prices are held constant. The income compensation function is generally written with p representing a vector of prices. One could, therefore, use the notation $\mu(p, 1/p^\circ, 1, m^\circ)$ to represent the fact that all prices but one are held constant. However, to simplify notation we subsume all other prices into the functional form of the income compensation function.

[2]One regularity condition is that both H_1^* and H_2^* , and either H_{12}^* or H_{21}^* must be continuous. In addition, the domain of integration G must be "simply connected." i.e., there must be "no holes" in the domain. More precisely, G is simply connected if, for every simple closed curve B in G, the region R formed of B plus its interior lies wholly in G. If the domain of integration is not simply connected, then the first two statements are equivalent and imply the third. But the converse is not necessarily true. See Kaplan (2:243-248) for a discussion of these equivalencies.

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